

Enrollment No: \_\_\_\_\_

Exam Seat No: \_\_\_\_\_

# C.U.SHAH UNIVERSITY

## Summer Examination-2017

**Subject Name: Mathematics-II**

**Subject Code: 4SC02MAT1**

**Branch: B.Sc. (All)**

**Semester: 2**

**Date: 09/05/2017**

**Time: 02:00 To 05:00**

**Marks: 70**

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

- Q-1          Attempt the following questions:          (14)**
- a)** The solution of the differential equation  $(D^2 - 2D + 1)y = 0$  is ..... (01)
- |                            |                           |
|----------------------------|---------------------------|
| (1) $c_1 e^x + c_2 e^{-x}$ | (3) $(c_1 + c_2 x)e^{-x}$ |
| (2) $(c_1 + c_2 x)e^x$     | (4) None of these         |
- b)** The particular integral of  $(D^2 + a^2)y = \sin ax$  is ..... (01)
- |                             |                             |
|-----------------------------|-----------------------------|
| (1) $-\frac{x}{2a} \cos ax$ | (3) $-\frac{ax}{2} \cos ax$ |
| (2) $\frac{x}{2a} \cos ax$  | (4) $\frac{ax}{2} \cos ax$  |
- c)** When we put  $z = \log x$  in a homogeneous linear differential equation, the value of  $x^2 \frac{d^2 y}{dx^2}$  is ..... (01)
- |  |  |
|--|--|
| (1) $z^2 \frac{d^2 y}{dz^2}$                   | (3) $\frac{d^2 y}{dz^2} - \frac{dy}{dz}$ |
| (2) $z^2 \frac{d^2 y}{dz^2} - z \frac{dy}{dz}$ | (4) $\frac{d^2 y}{dz^2} + \frac{dy}{dz}$ |
- d)** The particular integral of the differential equation  $(D^2 - 3D + 2)y = e^{5x}$  is ... (01)
- |                           |                          |
|---------------------------|--------------------------|
| (1) $e^{5x}$              | (3) $\frac{1}{6} e^{5x}$ |
| (2) $\frac{1}{12} e^{5x}$ | (4) $\frac{1}{4} e^{5x}$ |
- e)**  $\frac{1}{D-m} Q$  is equal to ..... (01)
- |                                |                                |
|--------------------------------|--------------------------------|
| (1) $e^{mx} \int Q dx$         | (3) $e^{-mx} \int Q dx$        |
| (2) $e^{-mx} \int Q e^{mx} dx$ | (4) $e^{mx} \int Q e^{-mx} dx$ |
- f)** The complex conjugate of  $\frac{i}{1-i}$  is ..... (01)



- (1)  $\frac{-i}{1+i}$  (3)  $\frac{i-1}{2}$   
 (2)  $\frac{1-i}{i}$  (4) None of these
- g) Real part of  $\cosh z$  is ..... (01)  
 (1)  $\cosh x \cos y$  (3)  $\cos h x \sin y$   
 (2)  $\sinh x \sin y$  (4)  $\sinh x \cos y$
- h) If  $z = \cos \theta + i \sin \theta$ , then  $\sin n\theta = \dots\dots\dots$  (01)  
 (1)  $\frac{z^n + z^{-n}}{2}$  (3)  $\frac{z^n - z^{-n}}{2i}$   
 (2)  $\frac{z^n - z^{-n}}{2}$  (4) None of these
- i) If  $x + iy = \sqrt{2} + 3i$ , then  $x^2 + y^2$  is ..... (01)  
 (1) 7 (3) 13  
 (2) 5 (4)  $\sqrt{2} + 3$
- j) The real part of  $(\sin x + i \cos x)^5$  is ..... (01)  
 (1)  $-\cos 5x$  (3)  $\sin 5x$   
 (2)  $-\sin 5x$  (4)  $\cos 5x$
- k) General equation to the cone which passes through the axes is ..... (01)  
 (1)  $ax^2 + by^2 + cz^2 = 1$  (3)  $fyx + gzx + hxy = 1$   
 (2)  $ax^2 + by^2 + cz^2 = 0$  (4)  $fyx + gzx + hxy = 0$
- l) The equation of the enveloping cone can be written as: (01)  
 (1)  $S = T^2$  (3)  $T = S_1$   
 (2)  $SS_1 = T^2$  (4) None of these
- m) Guiding curve of a right circular cylinder is ..... (01)  
 (1) ellipse (3) pair of straight lines  
 (2) circle (4) any closed curve
- n) The equation  $\frac{x^2}{2} - \frac{y^2}{3} = z$  represents: (01)  
 (1) cylinder (3) ellipsoid  
 (2) hyperboloid (4) paraboloid

Attempt any four questions from Q-2 to Q-8

**Q-2 Attempt all questions (14)**

- a) Find the equation of cone whose vertex is  $(\alpha, \beta, \gamma)$  and base  $ax^2 + by^2 = 1, z = 0$ . (05)
- b) Describe and sketch the conicoid  $\frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{9} = 1$ . (05)
- c) Find the equation of a cylinder whose generating lines have the direction cosine  $(l, m, n)$  and which passes through the circle  $x^2 + z^2 = a^2, y = 0$ . (04)

**Q-3 Attempt all questions (14)**

- a) Find the equation of the enveloping cylinder of the sphere  $x^2 + y^2 + z^2 = 25$ , whose generators are parallel to the line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ . (05)
- b) Prove that the equation  $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$  (05)



represents a cone whose vertex is  $\left(-\frac{7}{6}, \frac{1}{3}, \frac{5}{6}\right)$ .

c) Define:  $\log(x + iy)$ . Determine  $\log(1 - i)$ . (04)

**Q-4** **Attempt all questions** (14)

a) Prove that the  $n^{\text{th}}$  root of unity are in a geometric progression. Also show that their sum is zero. (05)

b) Separate real and imaginary parts of  $\tan(x - iy)$ . (05)

c) Solve:  $y'' + 16y = x^4 + e^{3x} + \cos 3x$ . (04)

**Q-5** **Attempt all questions** (14)

a) Solve:  $(x^2 D^2 - 3xD + 4)y = x^2$ , given that  $y(1) = 1$  and  $y'(1) = 0$ . (05)

b) Solve:  $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 8y = x^2$ . (05)

c) Solve:  $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-3x}$ . (04)

**Q-6** **Attempt all questions** (14)

a) State and prove De-Moivre's theorem. (05)

b) Prove that  $\sinh^{-1}(z) = \ln(z + \sqrt{z^2 + 1})$ . (05)

c) Expand  $\cos^5 \theta$  in a series of cosines of multiples of  $\theta$ . (04)

**Q-7** **Attempt all questions** (14)

a) Solve:  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$ . (05)

b) Prove that the cones  $ax^2 + by^2 + cz^2 = 0$  and  $\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 0$  are reciprocal. (05)

c) Prove that  $\cos 6\theta = 32 \cos^6 \theta - 48 \cos^4 \theta + 18 \cos^2 \theta - 1$ . (04)

**Q-8** **Attempt all questions** (14)

a) Solve the simultaneous equations  $\frac{dx}{dt} + 2y + \sin t = 0$ ,  $\frac{dy}{dx} - 2x - \cos t = 0$  given that  $x = 0$  and  $y = 1$  when  $t = 0$ . (05)

b) Solve:  $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10 \left(x + \frac{1}{x}\right)$ . (05)

c) Identify the surface given by  $9x^2 + 4y^2 - 9z^2 - 18x - 8y - 18z = 32$ . (04)

